

Section - A

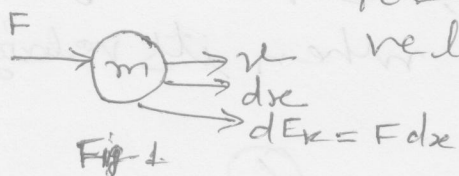
- Ans 1 (i) c (ii) b (iii) b (iv) b (v) c (vi) a (vii) a  
 (viii) a (ix) d (x) d

Section - BAns 2. Frame of Reference

The motion of a body can be described with respect to some well defined coordinate system. The coordinate system is known as frame of reference. There are two types of frame of reference:

- (i) Inertial or unaccelerated frames: - In this frame bodies obey Newton's laws of motion. In this frame a body not acted upon by external force, is at rest or moves with a constant velocity. The laws of physics will be same for all observers in this frame of reference or the laws of mechanics are the same in all inertial frame of reference.
- (ii) Non-inertial frames: - In this frame Newton's laws are not valid and a body, not acted upon by an external force is accelerated. This is also called a rotating frame.

Einstein's Mass-Energy Equivalence: - Let us consider a particle of mass  $m$  acted upon by a force  $F$  in the same direction as its velocity  $v$  and the force displaces the body through a distance  $dx$ . Then increase in the K.E. ( $dE_k$ ) of



body is equal to the <sup>(2)</sup> work done ( $F dx$ ) as shown in Fig. 1; Now the force is defined as rate of change of momentum i.e.  $F = \frac{dp}{dt}$

$$\text{Thus, } F = \frac{d(mu)}{dt} = m \frac{du}{dt} + u \frac{dm}{dt} \quad \text{--- (1)}$$

Because according to theory of relativity both mass and velocity are variable

$$\text{Now } dE_k = F dx = m \frac{du}{dt} dx + u \frac{dm}{dt} dx \quad [\text{From eqn. 1}]$$

$$\text{or } m \frac{dx}{dt} du + u \frac{dx}{dt} dm$$

$$\text{or } m v du + u^2 dm \quad [ \because \frac{dx}{dt} = v ] \quad \text{--- (2)}$$

Now, according to the variation of mass with velocity  $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$  --- (3)

$$\text{or } m^2 = \frac{m_0^2}{\left(1 - \frac{v^2}{c^2}\right)} = \frac{m_0^2}{\frac{c^2 - v^2}{c^2}} = \frac{m_0^2 c^2}{c^2 - v^2} = m_0^2 c^2 = m^2 (c^2 - v^2)$$

$$= m^2 c^2 - m^2 v^2 \quad \text{or} \quad \boxed{m^2 c^2 = m_0^2 c^2 + m^2 v^2}$$

Differentiating above eqn. we can get

$$c^2 2m dm = m^2 2v dv + v^2 2m dm$$

$$\text{or } c^2 dm = m v dv + v^2 dm \quad \text{--- (4)} \quad [ \because c \text{ and } m_0 \text{ are const.} ]$$

on comparing eqns (2) and (4) we can write

$$dE_k = c^2 dm \quad \text{or} \quad dE_k \propto dm \quad \text{--- (5)}$$

Thus, a change in KE ( $dE_k$ ) is directly proportional to a change in mass  $dm$ . Therefore, integrating eqn (5)

$$\int_0^{E_k} dE_k = \int_{m_0}^m c^2 dm \quad \text{or} \quad E_k = \int_0^{E_k} dE_k = c^2 \int_{m_0}^m dm = c^2 (m - m_0)$$

Because, when a body is at rest, its velocity is zero, KE is zero and  $m = m_0$ ; and when its velocity is  $v$  its mass becomes  $m$ .

$$E_k = mc^2 - m_0 c^2 \quad \text{--- (6)}$$

The above eqn. <sup>(3)</sup> is relativistic formula for K.E. When the body is at rest, energy stored in the body is  $m_0 c^2$  which is called the rest mass energy.

Thus, the total energy  $E$  of the body is the sum of K.E. and rest mass energy i.e.

$$E = E_k + m_0 c^2 = (m c^2 - m_0 c^2) + m_0 c^2 = m c^2$$

or  $\boxed{E = m c^2}$  ----- (7)

This eqn. is known as Einstein's mass, energy equivalence and states that mass may appear as energy and energy as mass.

OR

Let  $l_0$  is the length of the rod in the frame in which it is at rest and  $S'$  is the frame which is moving with a speed  $0.8c$  in a direction making an angle  $60^\circ$  with  $x$ -axis. The components of  $l_0$  along and perpendicular to the direction of motion are  $l_0 \cos 60^\circ$  and  $l_0 \sin 60^\circ$ , respectively.

Now length of the rod along the direction of motion =  $l_0 \cos 60^\circ \sqrt{1 - \frac{(0.8c)^2}{c^2}} = \frac{l_0}{2} \times 0.6 = \underline{0.3l_0}$

Length of the rod perpendicular to the direction of motion =  $l_0 \sin 60^\circ = \frac{\sqrt{3}}{2} l_0$

Length of the moving rod

$$l = [(0.3l_0)^2 + \left(\frac{\sqrt{3}}{2} l_0\right)^2]^{1/2} = 0.916 l_0$$

$$\% \text{ contraction} = \frac{l_0 - 0.916 l_0}{l_0} \times 100 = \underline{8.4\% \text{ Ans}}$$

3. The formation of Newton's rings is a special case of interference in an air film of variable thickness. When a plano-convex lens of large focal length is placed on a plane glass plate, the thin film of air is formed between the lower surface of the lens and the upper surface of the glass plate. When a monochromatic light is allowed to fall normally on the film, then circular fringes are observed. In reflected light the centre of the circular fringes are dark followed by alternatively bright and dark circular rings.

The experimental arrangement to observe Newton's ring is shown in Fig., where

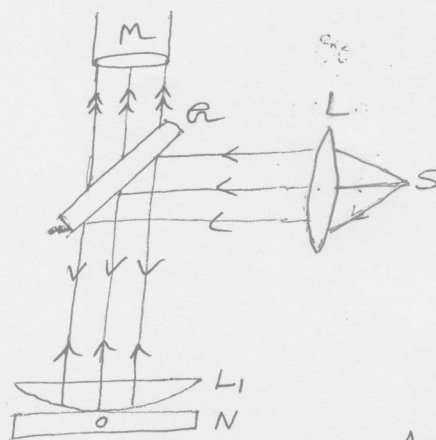


Fig. Expt. arrangement to observe Newton's ring.

S = an extended monochromatic source

L = convex lens.

G = glass plate

L<sub>1</sub> = plano-convex lens of large focal length

N = plane glass plate

M = microscope

A sodium lamp S is placed at the focus of a convex lens L. The horizontal parallel rays after the lens fall on a glass plate G inclined at 45°. The rays are partly reflected from the inclined glass plate and fall normally on the plano-convex lens of large focal length L<sub>1</sub> placed over the plane glass plate N.

The thin air film is formed between the plano-convex lens L<sub>1</sub> and the glass plate N around the point of contact, O. The interference takes place between the rays reflected from the upper and lower surfaces of the

⑤

film and are viewed with a microscope  $M$  focussed on the air film.

The thin air film formed between the curved surface of the plano-convex lens and plane glass plate is of wedge shape. So the path difference between the interfering rays in ~~the~~ reflected light will be  $2\mu t \cos(r+\theta) + \lambda/2$ , where

$\mu$  = refractive index of the thin film  
 $t$  = thickness " " " "  
 $r$  = angle of reflection  
 $\theta$  = angle of wedge  
 $\lambda$  = wave-length of the used monochromatic light.

For, normal incidence  $r=0$  and in this case the wedge-angle  $\theta$  = very small  $\approx 0$ .

$\therefore$  The path difference =  $2\mu t + \lambda/2$  where  $\mu=1$  for air.

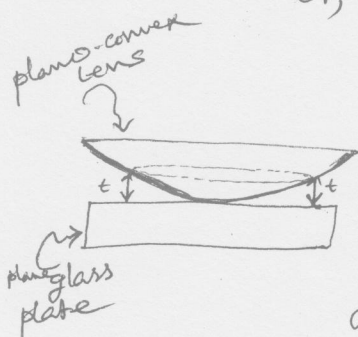
Now for constructive interference / bright ring

$$2\mu t + \lambda/2 = n\lambda \quad \text{and for}$$

destructive interference / dark ring

$$2\mu t + \lambda/2 = (2n+1)\lambda/2$$

$$\text{or, } 2\mu t = n\lambda.$$



Newton's rings are circular because the air film formed is wedge shaped and loci of the points of equal thickness  $t$  are circles concentric with the point of contact.

At the point of contact of plano-convex lens and glass plate, the actual path difference between the two interfering rays is zero but because one of the interfering rays is reflected from glass plate which is a denser medium, the effective path difference is  $\lambda/2$  or a phase change of  $\pi$ . Thus, at the centre, the condition of minimum intensity is satisfied and the central spot of the ring is dark.

3. or

Unit-II

(6)

It is optically flat glass plate on which large number of equidistant parallel lines are ruled by a fine diamond point. The space between the successive lines are transparent while the lines drawn are opaque to light.

Generally, there are 10,000 to 15,000 lines per inch in a plane transmission grating.

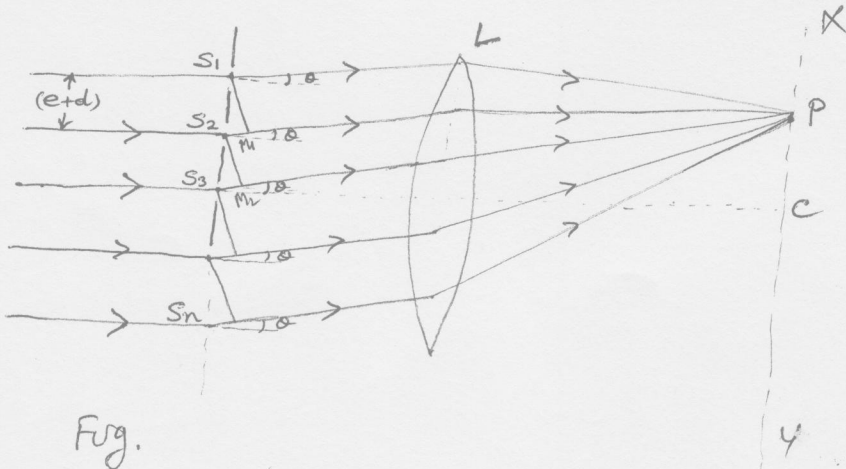


Fig.

Since a plane diffraction grating is a  $N$ -slit arrangement, the diffraction pattern due to it will be combined diffraction effect of all such slits. Let a plane wavefront of monochromatic light be incident normally on  $N$ -parallel slits of the grating. Each point in the slits then sends out secondary wavelets in all directions. Let  $e$  be the width of each slit and  $d$  be the separation between any two consecutive slits, then  $(e+d)$  is known as the grating element. The diffracted rays from each slit are focussed at a point  $P$  on the screen  $XY$  with the help of a convex lens  $L$ . Let  $S_1, S_2, S_3, \dots$  be the middle point of each slit and  $S_1M_1, S_2M_2, \dots$  be the perpendiculars drawn as shown in the figure. The wave diffracted from each slit is equivalent to a single wave of amplitude

$$R = \frac{A \sin \alpha}{\alpha}$$

The path difference between the waves from  $S_1$  &  $S_2$  is

$$S_2M_1 = (e+d) \sin \theta$$

The path difference between the waves from  $S_2$  &  $S_3$  is

$$S_3M_2 = (e+d) \sin \theta$$

The path difference between the waves from  $S_{n-1}$  &  $S_n$  is  $S_n M_{n-1} = (e+d) \sin \theta$

Thus, the path difference between all the consecutive waves is the same and equal to  $(e+d) \sin \theta$ . The corresponding phase difference

$$= \frac{2\pi}{\lambda} (e+d) \sin \theta$$

$$= 2\beta \text{ (say)}$$

Thus, the resultant amplitude at P is the resultant amplitude of N waves, each of amplitude R and common phase difference  $(2\beta)$ .

$\therefore$  The resultant amplitude at P is given by

$$R' = \frac{R \sin \frac{2N\beta}{2}}{\sin \frac{2\beta}{2}}$$

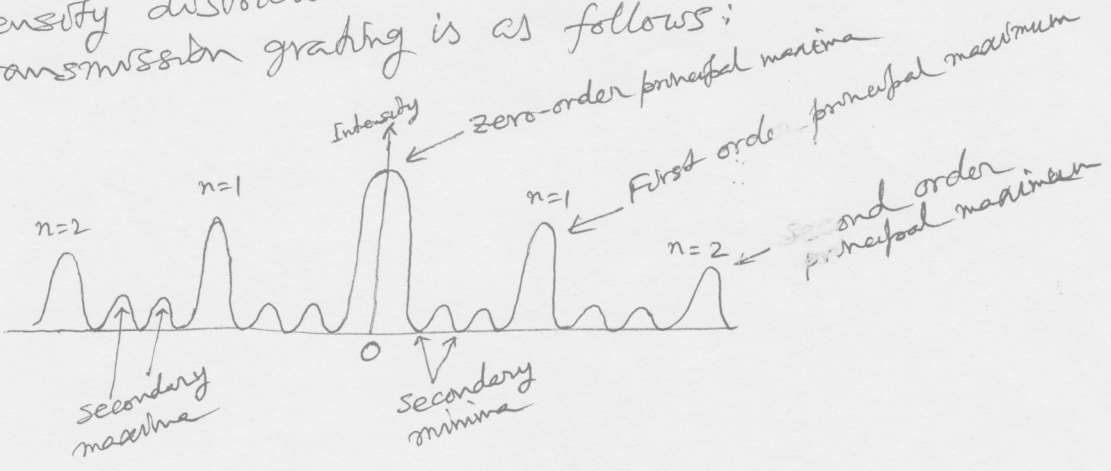
$$\text{or, } R' = R \frac{\sin N\beta}{\sin \beta} = \frac{A \sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$$

The resultant intensity at P is given by

$$I = R'^2 = \frac{A^2 \sin^2 \alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2 \beta} \quad \text{--- (1)}$$

$\xleftarrow{\text{intensity pattern due to a single slit}}$       $\xleftarrow{\text{distribution of intensity due to interference from all the N slits}}$

The intensity distribution in diffraction pattern due to a plane transmission grating is as follows:



Principal Maxima

From eqn. (1) it is clear that the intensity will be

maximum, when  $\sin\beta = 0$

$$\text{or, } \beta = \pm n\pi, \quad n = 0, 1, 2, 3, \dots$$

$$\begin{aligned} \text{Now } \lim_{\beta \rightarrow \pm n\pi} \frac{\sin N\beta}{\sin\beta} &= \lim_{\beta \rightarrow \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)} \\ &= \lim_{\beta \rightarrow \pm n\pi} \frac{N \cos N\beta}{\cos\beta} = N \text{ sec}^2 \end{aligned}$$

Now from ①

$$I = \frac{A^2 \sin^2 \alpha}{\alpha^2} \cdot N^2$$

$\therefore$  The condition for principal maxima is

$$\sin\beta = 0 \quad \text{or, } \beta = \pm n\pi.$$

$$\text{or, } \frac{\pi}{\lambda} (e+d) \sin\theta = \pm n\pi$$

$$\text{or, } (e+d) \sin\theta = \pm n\lambda$$

For  $n=0$ , we get  $\theta=0 \rightarrow$  gives zero order principal max.  
 $n=1, 2, 3, \dots$  gives the direction of first, second, third... order principal maxima.

Minima

The intensity is minimum, when

$$\sin N\beta = 0 \quad \text{but } \sin\beta \neq 0$$

$$\Rightarrow N\beta = \pm m\pi$$

$$\text{or, } N \cdot \frac{\pi}{\lambda} (e+d) \sin\theta = \pm m\pi$$

$$\text{or, } N \cdot (e+d) \sin\theta = \pm m\lambda$$

where  $m$  can take all integral values except  $0, N, 2N, 3N, \dots$

because for these values of  $m$ ,  $\sin\beta = 0$  which gives the positions of principal maxima.

For secondary maxima, to find out the condition we have to differentiate eqn ① with respect to  $\beta$  and equated to zero, i.e.

$$\frac{dI}{d\beta} = 0$$

To determine the wavelength of light we have to use the grating eqn. i.e. eqn. of  $n$ th order principal maxima for normal incidence  $(e+d) \sin\theta = n\lambda$   
 where  $(e+d) =$  grating element, known for a grating spectrum.



Unit - III (9)

Ans 4:- Given: energy of the lamp = 1000 watts  
= 1000 Joules/sec

$$\text{Area illuminated} = 4\pi r^2 = 4\pi(2)^2$$
$$= 16\pi \text{ m}^2$$

Therefore, energy radiated per unit area per second =  $\frac{1000}{16\pi}$

Hence from Poynting theorem,

$$|S| = |E \times H| = EH = \frac{1000}{16\pi} \text{ --- (1)}$$

$$\text{and } \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.72 \text{ --- (2)}$$

multiplying eqn (1) by (2), we get

$$E^2 = \frac{1000}{16\pi} \times 376.72$$

$$\boxed{E = 48.87 \text{ Volt/m}} \quad \underline{\underline{\text{Ans}}}$$

OR

Maxwell's equations in differential form can be given as

$$\text{div } D = \rho; \quad \nabla \cdot D = \rho \text{ --- (1)}$$

$$\text{div } B = 0; \quad \nabla \cdot B = 0 \text{ --- (2)}$$

$$\text{curl } E = -\frac{\partial B}{\partial t}; \quad \text{or } \nabla \times E = -\frac{\partial B}{\partial t} \text{ --- (3)}$$

$$\text{curl } H = J + \frac{\partial D}{\partial t} \text{ or } \nabla \times H = J + \frac{\partial D}{\partial t} \text{ --- (4)}$$

The propagation of electromagnetic waves results

in the transposition of energy from one place to other.

Taking scalar product of eqn (3) with  $H$  and eqn (4) with  $E$ , we get.

$$H \cdot \text{curl } E = -H \cdot \frac{\partial B}{\partial t} \quad \text{--- (5)}$$

$$\text{and } E \cdot \text{curl } H = E \cdot J + E \cdot \frac{\partial D}{\partial t} \quad \text{--- (6)}$$

Subtracting eqn. (6) from eqn (5), we get.

$$\begin{aligned} H \cdot \text{curl } E - E \cdot \text{curl } H &= -H \cdot \frac{\partial B}{\partial t} - E \cdot \frac{\partial D}{\partial t} - E \cdot J \\ &= \left( H \cdot \frac{\partial B}{\partial t} + E \cdot \frac{\partial D}{\partial t} \right) - E \cdot J \quad \text{--- (7)} \end{aligned}$$

Now using the vector identity,

$$\text{div}(E \times H) = H \cdot \text{curl } E - E \cdot \text{curl } H$$

The eqn (7) can be written as

$$\text{div}(E \times H) = - \left( H \cdot \frac{\partial B}{\partial t} + E \cdot \frac{\partial D}{\partial t} \right) - E \cdot J \quad \text{--- (8)}$$

Now using the relation  $B = \mu H$ ; and  $D = \epsilon E$ ; for a linear medium, eqn (8) becomes

$$\begin{aligned} \text{div}(E \times H) &= - \left( H \cdot \frac{\partial (\mu H)}{\partial t} + E \cdot \frac{\partial (\epsilon E)}{\partial t} \right) - E \cdot J \\ &= - \left\{ \frac{1}{2} \frac{\partial (\mu H^2)}{\partial t} + \frac{1}{2} \epsilon \frac{\partial (E^2)}{\partial t} \right\} - E \cdot J \\ &= - \left\{ \frac{1}{2} \frac{\partial (H \cdot \mu H)}{\partial t} + \frac{1}{2} \frac{\partial (E \cdot \epsilon E)}{\partial t} \right\} - E \cdot J \\ &= - \left\{ \frac{1}{2} \frac{\partial (H \cdot B)}{\partial t} + \frac{1}{2} \frac{\partial (E \cdot D)}{\partial t} \right\} - E \cdot J \end{aligned}$$

$$\text{div}(E \times H) = - \frac{\partial}{\partial t} \left[ \frac{1}{2} (E \cdot D + H \cdot B) \right] - E \cdot J \quad \text{--- (9)}$$

Integrating eqn (9) over a volume  $v$  bounded by a surface  $S$ , we get.

$$\int_V \text{div}(E \times H) dV = - \int_V \left\{ \frac{\partial}{\partial t} \cdot \frac{1}{2} (E \cdot D + H \cdot B) \right\} dV - \int_V E \cdot J \cdot dV$$

$$\text{or } \int_S (E \times H) \cdot ds = - \frac{\partial}{\partial t} \int_V \frac{1}{2} (E \cdot D + H \cdot B) dV - \int_V J \cdot E \cdot dV$$

$$\text{or } - \int_V J \cdot E \cdot dV = \frac{\partial}{\partial t} \int_V \frac{1}{2} (E \cdot D + H \cdot B) dV + \int_S (E \times H) \cdot ds$$

I<sup>st</sup> term
II<sup>nd</sup> term
III<sup>rd</sup> term (10)

1. The I<sup>st</sup> term of eqn (10)  $-\int_V J \cdot E \cdot dV$  represents the rate of transfer of energy into the electromagnetic field due to the motion of charge.

2. The term  $\frac{d}{dt} \int_V \frac{1}{2} (E \cdot D + H \cdot B) dV = U_e + U_m = U_{\text{total}}$  and it represents the rate of E.M.E. stored.

3. The term  $\int_S (E \times H) \cdot ds$  represents the amount of energy crossing per second through the closed surface.

The factor  $E \times H = S$  is called the Poynting vector. The eqn. (10) thus represents the law of conservation of energy.



## Unit-IV

1. Define conductivity of a material. Find out expressions for electrical conductivity of metal, intrinsic and extrinsic semiconductors.

Ans:

Conductivity of material: - The electrical conductivity ( $\sigma$ ) may be define as the quantity of electricity that flows in unit time per unit area of cross section of the conductor per unit potential gradient.  
 or Electrical conductivity is the capability of the solid to conduct electric charge under influence of an electric field. This is reciprocal to resistivity.

Electrical conductivity of metal: - Let us consider a rectangular block of length 'L' and cross-sectional area A (as shown in fig.). Let n be the concentration of free electrons available in it. Then total charge contained in the block is given as  $Q = Nq = nqAL$

$$\therefore \text{current } I = \frac{nqAL}{t} = nqAV_d$$



The current density  $J = \frac{I}{A} = nqV_d$  (where  $V_d = \frac{1}{t}$ ; drift velocity of electron in the solid) ————— ①

Using ohm's law  $I = V/R$  where  $R = \frac{\rho L}{A}$

$$= \frac{VA}{\rho L} = \sigma AE.$$

$$\Rightarrow \boxed{J = \sigma E} \text{ ————— ②}$$

From equation ① & ②, we have  $\sigma = nq \frac{V_d}{E} = nq\mu$

$$\Rightarrow \boxed{\sigma = nq\mu} \text{ (where } \mu \text{ is mobility of charge carrier)}$$

We see that conductivity is proportional to the concentration (n) of free electrons.

Conductivity of Semiconductor materials: - The conductivity of a semiconductor is different from a metal in the respect that in a semiconductor the charge carriers are electrons as well as holes. When an electric field E is applied to a semiconductor block, the current densities contributed due to the motion of electrons and holes are given by the expressions:  $J_n = qnV_n$  and  $J_p = qpV_p$

where q: charge of an electron (or a hole)

n: density of free electron

p: density of holes

$V_n$  &  $V_p$  :- drift velocity of free electrons

conductivity due to holes  $\sigma_p = \frac{J_p}{E} = q p \mu_p$ .

where  $\mu_n$  &  $\mu_p$  is the mobility of electrons & holes respectively

Total conductivity of a semiconductor  $\sigma = \sigma_n + \sigma_p$

$$\sigma = q [n \mu_n + p \mu_p]$$

Intrinsic Semiconductor :-  $n = p = n_i$

$$\sigma_i = q [n_i \mu_n + n_i \mu_p] = q n_i (\mu_n + \mu_p)$$

conductivity of Extrinsic Semiconductor :-

(i) In n-type semiconductor; the ~~total~~ electron concentration is much greater than the hole concentration, i.e.  $n \gg p$

$$\sigma_n \approx q n \mu_n$$

OR

Mobilities of electrons and holes in a sample of intrinsic germanium at 300K are  $0.36 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$  and  $0.17 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$ , respectively. If the conductivity of the specimen is 2.12 mho/m, calculate the forbidden energy gap.

Ans:

The formula for conductivity is  $\sigma_i = n_i e (\mu_n + \mu_p)$

$$\text{or } n_i = \frac{\sigma_i}{e (\mu_n + \mu_p)}$$

with (i)  $\sigma_i = 2.12 \text{ } \Omega^{-1} \text{ m}^{-1}$

(ii)  $e = 1.6 \times 10^{19} \text{ coulomb}$

(iii)  $\mu_p = 0.17 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$

(iv)  $\mu_n = 0.36 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$

Thus  $n_i = \frac{2.12}{1.6 \times 10^{19} \times (0.17 + 0.36)}$   
 $= 2.5 \times 10^{19} \text{ m}^{-3}$ .

But  $n_i = c T^{3/2} \exp\left[-\frac{E_g}{2k_B T}\right]$

with (i)  $c = 2 \left[ \frac{2\pi m k_B}{h^2} \right]^{3/2} = 4.83 \times 10^{21}$

(ii)  $T = 300 \text{ K}$ .

(iii)  $2k_B T = 0.052 \text{ eV}$

(iv)  $n_i = 2.5 \times 10^{19} \text{ m}^{-3}$

$\Rightarrow \exp\left[\frac{E_g}{2k_B T}\right] = \frac{c T^{3/2}}{n_i} = 10^6$

$\Rightarrow E_g = (2k_B T) \ln 10$   
 $= 0.72 \text{ eV}$ .

$E_g = 0.72 \text{ eV}$  Ans.

## Unit-V

2. What are the de Broglie matter waves? Explain in brief Davisson and Germer experiment and show that it provides direct evidence of de-Broglie's hypothesis.

**Ans:** The de Broglie Waves or Matter Waves

According to de Broglie, a matter particle having a mass  $m$  in moving with a velocity  $v$  must possess a matter wavelength equivalent to

$$\lambda = \frac{h}{mv} \quad \text{-----(1)}$$

where  $h$  is the universal Planck's constant. Louis de Broglie was led to this hypothesis by considering the special theory of relativity and quantum theory. Evidence for the matter waves was found in 1927 in two different laboratories. C.J. Davisson and L.H. Germer using a metal crystal as a reflection grating and G. P. Thomson employing a metal foil as a transmission grating showed that the electrons could be diffracted and thereby established both their wave particle nature and the quantitative validity of the de Broglie's hypothesis.

Since 1927 it has been shown that material particles other than electrons have wave properties: thus diffraction effects have been observed with hydrogen and helium nuclei and also with neutrons. There is a little doubt that the wave particle duality is a property of all forms of matter. However, it can be seen from Eq. (1) that with increasing mass, the wavelengths become shorter for a given velocity and so are increasingly difficult to detect

The major advantage of diffraction of electrons and neutrons have been utilized in the study of molecular and crystal structure. Further with the electron microscope, wherein de Broglie's concept of electron waves are involved, it has been possible to resolve objects as small as  $10\text{\AA}$  in size compared with a minimum of about 300 nm in an ordinary microscope.

de Broglie Wavelength:

We have for electromagnetic radiation,

$$\begin{aligned} E &= mc^2 \text{ and } E = hv \\ \therefore mc^2 &= hv = \frac{hc}{\lambda} \\ \lambda &= \frac{h}{mc} \quad \text{-----(2)} \end{aligned}$$

Equation (1) gives the expression for the wavelength of a photon wave that moves through a medium when photon travels with a velocity equal to velocity of light,

Similarly, any material particle having a mass  $m$  and moving with a velocity  $v$  must possess a de Broglie wavelength given by

$$\lambda = \frac{h}{mv}$$

de-Broglie wavelength in terms of kinetic energy

As de-Broglie waves are associated with particles that are moving with a measurable velocity  $v$ ,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

Characteristics of Matter Waves:

[i] From Eq. [1],  $\lambda \propto \frac{1}{m}$

Thus the wavelength of matter wave is inversely proportional to the mass of the particle. The larger the mass of the particle, the shorter will be the wavelength and vice versa.

(ii) From Eq. [1],  $\lambda \propto \frac{1}{v}$

Thus the matter wavelength varies inversely with the velocity of the particle. The greater the velocity of the particle, the smaller will be the matter wavelength and vice versa.

(iii) This is totally a new wave and cannot be equated to electromagnetic wave.

(iv) The velocity of matter wave depends on the velocity of matter particle, hence its velocity is not a constant whereas the velocity of electromagnetic wave is.

In 1927, Davisson and Germer predicted experimentally the electron waves predicted by de Broglie. Davisson and Germer were studying the scattering of electrons from a nickel target using an apparatus like that sketched in Fig.1. The energy of the electrons in the primary beam, the angle at which they are incident upon the target and the position of the detector could all be varied. The nickel target was subjected to such a high temperature treatment that the crystal was transformed into a group of crystals. In this case the reflection became anomalous and the reflected intensity showed striking maxima and minima instead of a continuous variation of scattered electron intensity with angle. The position of the maxima and minima observed depended upon the electron energy. Then, they suspected that the beam of electron might be diffracted from the crystals like X-rays. This shows that electrons behave like waves under certain circumstances. Typical polar graphs of electron intensity after the heat treatment are shown in Fig. 2.

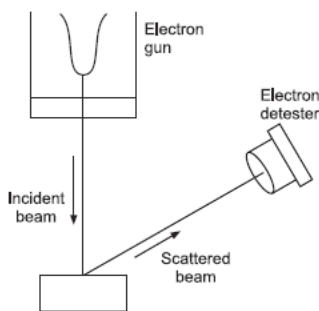


Fig. 1:

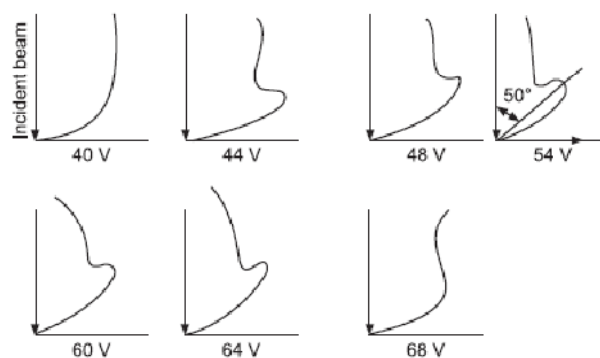


Fig. 2:

To verify whether de Broglie waves are responsible for the findings of Davisson and Germer, an analysis of the observation should be made. For the beam of electrons falling normally on the surface of the crystal, the current observed in detector is a measure of the intensity of the diffracted beam. Several curves were obtained for different voltage electrons when graphs were plotted between the colatitudes (angle between the incident beam and the beam entering the detector) which are shown in Fig. 3. It is observed that a bump begins to appear in the curve at 44 volt electrons. This bump moves upward for 54 volts at colatitudes of  $50^\circ$ . Above 54 volts the bump again diminishes. The bump at 54 volts offers the evidence for the existence of electron waves. The angles of incidence and scattering relative to the family of Bragg plane shown in Fig. 3 are both  $65^\circ$ . The spacing of the planes in this family, which can be measured by X-ray diffraction is 0.091 nm. The Bragg equation for maxima in the diffraction pattern is

$$n\lambda = 2d \sin\theta$$

Here  $d = 0.091$  nm,  $\lambda = 65^\circ$ . For  $n = 1$ , the de Broglie wavelength  $\lambda$  of the diffracted electrons is  $\lambda = 2(0.091)\sin 65^\circ = 0.165$  nm,

We use de Broglie formula to calculate the expected wavelength of the electrons. The electron kinetic energy of 54 eV is small compared with its rest energy  $m_0c^2$  of 0.51 MeV, so we can ignore relativistic considerations.

Since

$$K = \frac{1}{2}mv^2$$

The electron momentum  $mv$  is

$$mv = \sqrt{2mK} = 4.0 \times 10^{-24} \text{ kg.m/s}$$

The electron wavelength is therefore

$$\lambda = \frac{h}{mv} = 1.66 \times 10^{-10} \text{ m} = 0.166 \text{ nm}$$

is in excellent agreement with the observed wavelength. The Davisson-Germer experiment thus provides direct verification of de Broglie's hypothesis of the wave nature of moving bodies.

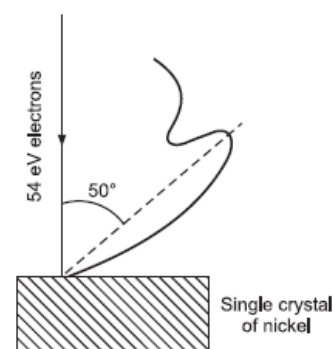


Fig. 3

OR

Calculate the de Broglie wavelength, if an electron is accelerated from rest through a potential difference  $V = 50$  Volt.

**Ans:**

The de-Broglie wavelength given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where  $h = 6.62 \times 10^{-34}$  J.s,  $m =$  mass of particle and  $v =$  velocity of particle.

The de-Broglie wavelength for accelerated charged particle through a potential volt is given by, Kinetic energy is

$$K = \frac{1}{2}mv^2 = eV$$

$$\Rightarrow eV = \frac{p^2}{2m}$$

$$\Rightarrow p = \sqrt{2meV}$$

Here,

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$V = 50 \text{ volt}$$

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.28}{\sqrt{V}} \text{ \AA}$$

$$\lambda = \frac{12.28}{\sqrt{50}} \text{ \AA} = 1.7366 \text{ \AA}$$